Després 60
Book of Abstracts

Anouk Nicolopoulos

Title : A few mathematical problems in fusion: a startup's perspective

Abstract : After a PhD working on electromagnetic wave propagation in tokamak plasmas with Bruno Després, and switching to the development of optimization algorithms, this eventually led me to work as a stellarator computational scientist in a fusion startup.
From optimizing current patterns to match target magnetic fields, to modeling the free-surface flow of liquid metals, applying machine learning to neutron shielding, and running topology optimization on reactor structures — I'll share a selection of math-related problems we've been tackling at Renaissance Fusion.
The aim is to give an idea of the variety of applied mathematics involved in fusion R&D, as seen from the inside of a startup.

Yann Brenier (CNRS, Université Paris-Saclay)

Title: Le système de Vlasov-Monge-Ampère et son application en astrophysique

Abstract: Le système de Vlasov-Monge-Ampère est la variante du système de Vlasov-Poisson où l'on remplace l'équation de Poisson par l'équation de Monge-Ampère, ce qui permet, au prix de la non-linéarité, de mieux traiter les secondes membres mesures en évitant les inconvénients de la fonction de Green du Laplacien. En cela le système de Vlasov-Monge-Ampère ressemble à l'électromagnétisme non-linéaire de Born-Infeld.
Il y a plusieurs façons d'obtenir le système de Vlasov-Monge-Ampère : par exemple comme la limite de la pénalisation des équations d'Euler des fluides incompressibles en partant de leur description géométrique à la Arnold ; la plus récente consiste à partir du principe des grandes déviations appliquées au mouvement d'un nuage brownien (travail en commun avec Luigi Ambrosio et Aymeric Baradat).
Pour le calcul numérique, l'algorithme géométrique de transport optimal introduit par Quentin Mérigot dans les années 2010 et généralisé à la dimension 3 par Bruno Lévy permet de réaliser des simulations à grande échelle de l'univers primitif (100 millions de particules où chaque particule représente un petit amas de galaxies).
L'exposé présentera un travail en commun avec Bruno Lévy (INRIA) et Roya Mohayahee (Institut d'Astrophysique de Paris).

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Borjan Geshkovski (Inria, LJLL)

Title: Transformers' forward pass is Frank-Wolfe in disguise

Abstract: Transformers—the neural network architectures underlying all large language models—can be viewed as interacting particle systems where time corresponds to layer depth. In the infinite-temperature limit, their dynamics reduce to an instantiation of the Frank-Wolfe algorithm for a quadratic objective, which allows us to establish convergence results under various model parameters. In discrete time, this yields a natural iterative scheme, while in continuous time, it leads to a singular ODE. We prove well-posedness of this ODE by exploiting the geometric structure of the vector field, which is given by a maximum of a linear form over Voronoi-like cells. Finally, we analyze the (actual) finite-temperature model, interpreting it as a Markov chain, and prove that it manifests metastable/slow motion.

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Francois Bouchut (LAMA, Université Gustave Eiffel)

Title: A two-speed relaxation system for Euler equations, and application to low Mach flows

Abstract: We introduce a relaxation system to approximate the solutions to the barotropic Euler equations. We show that the solutions to this two-speed relaxation model can be understood as viscous approximations of the solutions to the barotropic Euler equations under appropriate sub-characteristic conditions. Our relaxation system is a generalization of the well-known Suliciu relaxation system, and it is entropy satisfying. A Godunov-type finite volume scheme based on the exact resolution of the Riemann problem associated with the relaxation system is deduced, as well as its stability properties. In a second part, we show how the new relaxation approach can be successfully applied to the numerical approximation of low Mach number flows. We prove that the underlying scheme satisfies the well-known asymptotic-preserving property in the sense that it is uniformly (first-order) accurate with respect to the Mach number, and at the same time it satisfies a fully discrete entropy inequality. This discrete entropy inequality allows us to prove strong stability properties in the low Mach regime. At last, numerical experiments are given to illustrate the behaviour of our scheme.

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Eric Sonnendrücker (TU Munich)

Title: Structure preserving methods for the MHD and Vlasov-Maxwell equations

Abstract: We will review recently proposed structure preserving methods based on Finite Element Exterior Calculus or dual grids for solving the MHD equations based on a hybrid MHD-FEEC method and the Vlasov-Maxwell equations with a Particle in Cell method. The emphasis will be on the preservation of important invariants, in particular the divergence constraints and the hamiltonian structure.

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Frédéric Lagoutière (Lyon)

Title: Homogenization for modeling compressible fluid mixtures

Abstract: In this talk, I will summarize some works carried out in recent years with Didier Bresch, Cosmin Burtea and Pierre Gonin--Joubert on the homogenization of the Navier-Stokes equations for viscous compressible fluids in space dimension 1. A mixture of several fluids with different behaviors (pressure laws and viscosities) is considered to be the limit of a sequence of situations in which the fluids are separated, where the characteristic length of the pure-fluid zones tends to 0. For each element of the sequence, since the fluids are separate, everything is clear: there is no ambiguity about pressure nor viscosity. Of course, the state of such a fluid is highly oscillatory, but we obtain estimates of the boundedness of the oscillations that allow to show that some quantities (velocities, densities, volume fractions...) converge weakly towards the solutions of a well-known mixture model, the Baer and Nunziato model. I will illustrate the presentation with some numerical results, which may shed some light on the technical results.

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Frédérique Charles (LJK, UGA)

Title: Modeling internal energy exchange in sprays.

Abstract: In this talk, I will present two models describing heat exchange in gas-particle mixtures. The starting point is a purely kinetic model (coupling of two collisional equations), in which collisions between gas molecules and elements of the dispersed phase (e.g. dust particles) are described by a diffuse reflection mechanism.

In the first model, by adding a new variable to the particle density function, we impose conservation of total energy at the level of each collision. We then establish a formal asymptotics of the kinetic coupling when two small parameters tend to zero towards a Vlasov-Euler type system. This system then contains an explicit term describing the internal energy transfer between the two phases. This first part is the result of collaborative work with Laurent Desvillettes.

In a second model, the particle surface temperature is this time seen as a function of t and x, and its evolution is described by an equation allowing global conservation of the system's total energy. We then perform diffusive asymptotics, leading to a two-species cross-diffusion system. This second part is the result of collaborative work with Francesco Salvarani and Annamaria Massimini.

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M. A. Puscas (CEA Saclay)

Title: Sensitivity analysis for incompressible Navier-Stokes equations: uncertainty analysis and estimation of parameters

Abstract: This work proposes an efficient computational strategy for addressing uncertainty propagation problems related to the Navier–Stokes equations, which are coupled with a temperature equation based on a sensitivity analysis technique. Sensitivity analysis enables an examination of how the model response at a specific point is influenced by changes in the boundary conditions, including temperature, heat capacity, thermal conductivity, and thermal diffusivity, under the assumption of small variances in the input parameters. The variance can be estimated using a single simulation of the state and as many simulations of the sensitivity as uncertain parameters exist [2, 4]. The focus is on deriving and analyzing the stability of both the state and sensitivity equations [1, 3]. Applications include canonical benchmark problems such as channel flow, natural convection in a square cavity, and lid-driven cavity flow. These applications demonstrate the capacity of sensitivity methods to provide accurate variance estimates and confidence intervals. The open-source code TrioCFD [5] is utilized to simulate the state equations, and a specific module has been developed for the sensitivity equations.

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Camilla Fiorini (M2N, CNAM)

Titre : Hybrid autoencoder/Galerkin approach for nonlinear reduced order modelling

Résumé : This study introduces a nonlinear reduced order model (ROM) for fluid dynamics, which combines proper orthogonal decomposition (POD) with deep learning error correction. Our approach merges the interpretability and physical adherence of classical POD Galerkin ROMs with the predictive capabilities of deep learning. The hybrid model addresses errors within and outside the POD subspace. Firstly, POD generates part of the reduced state, complemented by an autoencoder compressing only the unretained POD modes. Thus, the most energetic modes are computed interpretably, while the least energetic are handled with a superior reduction method. Secondly, the time integration employs a hybrid neural Ordinary Differential Equation (neural ODE). A POD ROM estimates part of the dynamics, and a deep learning model corrects its error. Using Neural ODE aligns the model with underlying physics for enhanced stability and accuracy. The proposed method differs from current hybrid methods operating solely in the POD subspace and using Mori-Zwanzig time dependency, posing potential initialisation issues. Our model is applied to the viscous Burgers' equation, the parametric circular cylinder flow, and the fluidic pinball test case. Accuracy and numerical complexity are compared to classical POD Galerkin ROMs, fully data-driven models, and concurrent hybrid methods.

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Jean-David Benamou (Inria Paris)

Titre : Rocquencourt Bat. 13

Resume : On reviendra sur la méthode de decomposition de domaine inventée par Bruno pendant sa thèse et son influence et impact sur la carrière de l'orateur.
Ainsi que sur quelques autres vieux dossiers.

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Hélène Barucq (Inria Sud-Ouest)

Title: To use or not to use the Trefftz method, that's one of Bruno's many spicy questions

Abstract: In this presentation, we will return to the solution of the Helmholtz equation in large heterogeneous domains, in an attempt to answer the question: is there a preferred numerical method, whatever the simulation objective? After describing the mathematical problem in context, we'll explain the difficulties that may be encountered, depending on the intended application and the computing resources available. We will then present a set of results using finite element methods that concentrate the essential of the calculations on the mesh skeleton, with the idea of comparing a Trefftz method with an HDG method.

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Emmanuel Labourasse (CEA, DAM, DIF)

Two ways to enforce constraints in Finite-Volume schemes for Lagrangian hyperbolic systems

This talk is devoted to Lagrangian finite-volume schemes. This theme occupies a significant place in Bruno Després' scientific production. We approach it from the perspective of adding constraints applied to discrete solutions of the system of Euler equations or hyperelasticity. In the first part, the constraints are induced by a modification of the discrete entropy. This allows stabilizing the numerical method in the case of polygonal meshes. In the second part, the numerical method is reinterpreted as a minimization problem to be able to apply a connection constraint between fluid and solid domains. In both cases, the properties of the original numerical methods (without constraints) are preserved. The first part is a joint work with Bruno Després and the second part with Teddy Chantrait, Nicolas Chevaugeon, Stéphane Del Pino and Alexandre Gangloff.

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